sources, will not be proposed as problems for solution in the MONTHLY. In so far as possible, however, the editors will be glad to assist members of the Association with their difficulties in the solution of such problems.]

2908. Proposed by L. E. DICKSON, University of Chicago.

If f is a homogeneous polynomial in n variables and H is its Hessian determinant, prove that the Hessian of f^2 is cHf^n, where c is a constant.

2909. Proposed by J. S. STROMS, University of Maine.

A pinochle pack contains 48 cards, eight each of aces, kings, queens, jacks, tens, and nines. When three play, they are distributed by giving fifteen to each player and leaving three in the "kitty." When one holds two jacks of diamonds and two queens of spades in the same hand he has what is called double pinochle. When he holds eight aces he wins the game with 1000 points. What are the chances (*a*) of getting double pinochle; (*b*) of getting eight aces?

2910. Proposed by DANIEL KRETH, Wellman, Iowa.

The segments formed on the base of a triangle by the perpendicular from the opposite vertex are m and n. The product of the other two sides is p. Compute the two unknown sides and give a simple construction for the triangle.

2911. Proposed by D. A. ABRAMS, Chicago, Ill..

In the analysis of stresses in a flat slab of reinforced concrete, the following integral arises: $\int_0^x \int_0^y \int_0^z dxdydz$, where the upper limits for x, y, z are b, a, and $wb^3y^3(a-y)^3/[a^3x^3(b-x)^3 + b^3y^3(a-y)^3]$. Evaluate this integral.

2912. Proposed by T. W. JACKSON, Jamestown College, N. D.

Given c, the chord of a circle, determine r, the radius, so that $3c$ is equal to the major arc of the circle.

2913. Proposed by PAUL CAPRON, U. S. Naval Academy.

Given

$$S(z) \equiv nz\left[1+\sum_1^\infty(-1)^p\frac{(n^2-1)(n^2-3^2)\cdots\{n^2-(2p-1)^2\}}{(2p+1)!}z^{2p}\right]$$

$$C(z) \equiv 1+\sum_1^\infty(-1)^p\frac{n^2(n^2-2^2)(n^2-4^2)\cdots\{n^2-(2p-2)^2\}}{(2p)!}z^{2p}$$

show that, for any value of n, $|\cos x| < 1$, $|\sin x| < 1$:

$$\sin(nx) \equiv S(\sin x) \equiv \sin\left(\frac{n\pi}{2}\right)C(\cos x) - \cos\left(\frac{n\pi}{2}\right)S(\cos x);$$

$$\cos(nx) = C(\sin x) \equiv \cos\left(\frac{n\pi}{2}\right)C(\cos x) + \sin\left(\frac{n\pi}{2}\right)S(\cos x).$$

2914. Proposed by HARRIS HANCOCK, University of Cincinnati.

If $a_1, a_2, \cdots, a_n$ are n positive integers, a_{ij} the greatest common divisor of a_i and a_j, d_m the greatest common divisor of all products of every m of these numbers $(m = 1, 2, \cdots, n-1)$, then is

$$\prod_{ij} a_{ij} = d_1d_2\cdots d_{n-1}; \quad (j > i;\ i = 1, 2, \cdots, n;\ j = 2, 3, \cdots, n).$$

In general show that this theorem is true if $A_1, A_2, \cdots, A_n$ are any functions integral in any number of variables, with rational integral coefficients, or with algebraic integral coefficients. [Remark: This is a generalized statement in positive rational integers of the following theorem of much importance in the theory of algebraic numbers and due to Dedekind (Dirichlet, *Zahlentheorie*, supplement XI): Let $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ be three moduls (Dedekind). Denote the greatest common divisor of $\mathbf{A}$ and $\mathbf{B}$ by $\mathbf{A}+\mathbf{B}$ and their product by $\mathbf{A}\cdot\mathbf{B}$. Dedekind proves that

$$(\mathbf{A}+\mathbf{B})(\mathbf{B}+\mathbf{C})(\mathbf{C}+\mathbf{A}) = (\mathbf{A}+\mathbf{B}+\mathbf{C})(\mathbf{AB}+\mathbf{BC}+\mathbf{CA}).$$

Denote the greatest common divisor of two moduls A_i and A_j, that is, $\mathsf{A}_i + \mathsf{A}_j$ by A_{ij}; and write

$$\mathsf{D}_1 = \mathsf{A}_1 + \mathsf{A}_2 + \cdots + \mathsf{A}_n, \qquad \mathsf{D}_2 = \mathsf{A}_1\mathsf{A}_2 + \mathsf{A}_1\mathsf{A}_3 + \cdots + \mathsf{A}_{n-1}\mathsf{A}_n,$$
$$\mathsf{D}_3 = \mathsf{A}_1\mathsf{A}_2\mathsf{A}_3 + \mathsf{A}_1\mathsf{A}_2\mathsf{A}_4 + \cdots + \mathsf{A}_{n-2}\mathsf{A}_{n-1}\mathsf{A}_n, \cdots$$
$$\mathsf{D}_{n-1} = \mathsf{A}_2\mathsf{A}_3 \cdots \mathsf{A}_n + \mathsf{A}_1\mathsf{A}_3 \cdots \mathsf{A}_n + \cdots + \mathsf{A}_1\mathsf{A}_2 \cdots \mathsf{A}_{n-1}.$$

Show that

$$\mathsf{A}_{12}\,\mathsf{A}_{13} \cdots \mathsf{A}_{n-1,\,n} = \mathsf{D}_1\mathsf{D}_2 \cdots \mathsf{D}_{n-1}.]$$

2915. Proposed by HARRIS HANCOCK, University of Cincinnati.

Determine x so as to satisfy the two congruences $3x^2 \equiv 0\ (3N)$, $x^3 + a \equiv 0\ (9N^2)$, where $a = N^2 \cdot n$, and the two integers N, n have no common factor, and neither contains a squared factor.

2916. Proposed by HARRIS HANCOCK, University of Cincinnati.

If p is any rational prime integer, and if $\alpha (\neq 1)$ is any root of $x^p - 1 = 0$, show that $p = \mathsf{P}_1 \cdot \mathsf{P}_2 \cdots \mathsf{P}_{p-1}$, where P_i $(i = 1, 2, \cdots, p - 1)$ are the ideals $(p, 1 - \alpha^i)$, which in turn may be reduced to principal ideals. [Remark: This is rather a good elementary example to show that an integer prime in one realm is factorable in a more extended realm.]

2917.

A parabola is rolled upon a fixed right line. Find the locus of (a) its vertex; and (b) its focus.

2918. Proposed by NATHAN ALTSHILLER-COURT, University of Okla.

Find planes which cut four given lines in four concyclic points.

2919. Proposed by V. M. SPUNAR, Chicago, Ill.

An equilateral hyperbola which touches a conic and is concentric with it is called a hyperbolic tangent to the conic. Being given two hyperbolic tangents to a conic, the arc of any third hyperbolic tangent which is intercepted by the first two subtends a constant angle at either focus of the given conic.

NOTES.

19. A Curve of Pursuit. Apropos of Note **10** (*1921*, 184; cf. *1921*, 278, 281), referring to a problem proposed by Lucas in 1877, a very similar problem is given as a worked out exercise in Bateman, *Differential Equations*, 1918, pp. 8–10. It is as follows: "Three boys running at the same speed u chase one another. A pursues B, B pursues C and C pursues A. Find a differential equation which will indicate the way in which the ratios of the sides of the triangle ABC vary." A footnote states that this problem is due to Professor Frank Morley. His differential equation is discussed in a paper by F. E. Hackett, "A numerical solution of the triangular problem of pursuit," in *The Johns Hopkins University Circular*, July, 1908, pp. 135–137.

A. H. WILSON.

20. A Problem in Investment. There has been inquiry concerning the following problem given in a less general form, with a reference to *Engineering News*, volume 48, 1902, pp. 362–363, in E. B. Skinner, *The Mathematical Theory of Investment*, Boston, 1913, p. 140.

Bonds are issued, N in number, of a face value of A each, bearing interest at rate r. At the end of a years and at the end of each year thereafter a certain number of these bonds is to be redeemed at a price which bears the ratio R to the face value A. How many bonds must be redeemed each year in order that the whole issue shall be paid for at the end of n years, and that the sum of the interest